Examination of finite element analysis and experimental results of quasi-statically loaded acetal copolymer gears

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Abstract

An elastic-plastic finite element analysis of the quasi-static loading of two acetal copolymer gears in contact is preformed. Load verses rotation of the gear set is compared to actual experimental results. The gear geometry is modeled by plane strain elements with variable thickness between the rim and web. Gear tooth failure is modeled by both deactivating and separating elements when the tensile strength is exceeded. Failure in the tooth root is best modeled by a nonlinear approach using separating elements.

1. Introduction

The primary functionality of gears is to transmit energy, and rotary or linear motion. The design for the kinematics of a gear set is geometrically controlled if it is a rigid body. However, for real materials the choice of material selection becomes critical in the design.

As more and more gears are being manufactured out of engineering polymers and being used in more demanding mechanical devices, the need to accurately predict their mechanical behavior is necessary. That is, there is a need to optimize the gear design prior to undergoing the expense of building a tool and testing them.

Traditionally, the gear designer had a limited number of analytical tools, as well as his own experience, to achieve this. One of the most recognized analytical tool's that was developed for the prediction of gear tooth strength was developed by Wilfred Lewis [4]. His method predicted a safe transmitted load on the gear tooth. It should be noted that this method has been modified throughout the years and is still in use by the industry today. Other elaborate techniques have been developed to determine the stress field in the gear teeth. Baronet and Tordion [3] used conformal mapping based on a transformation function given by Aida and Terauchi [1].

With the advent of the computer age, variational methods have made their way into the design engineers toolkit. The major reason is that they have become more user friendly and lend themselves to be very flexible in solving many complex engineering problems.

However, the uses of finite element analysis (FEA) methods are mostly used on materials that behave linear elastically. For metals, this assumption gives satisfactory results. Unfortunately polymer materials do not behave linear elastically. Only in a small range do they exhibit this type of behavior. The gear designer is greatly limited by using a linear elastic approach if he is to optimize his gear design out of polymer materials.

In this paper we perform an elastic-plastic finite element analyses for two acetal copolymer spur gears in mesh under a quasi-static load. We then compare these results to actual experimental data. As will be seen, depending on the approach used, the analyses give different results.

2. Test Specimens & Test Method

Two acetal copolymer spur gears were selected as test specimens. The geometry of the gear teeth was based on the American Gear Manufacturers Association (AGMA) standard: Tooth proportions for Plastic Gears [3].

These gears were molded using a diaphragm gate to eliminate any weld lines that would create a weak connection in the material. The specifications for the test gears used are provided in Table 1.

Table 1

Basic Specification Data:	
Number of Teeth	40
Diametrial pitch	20
Standard pressure angle	20
Tooth form	AGMA PT1
Standard addendum	.0500
Standard whole depth	.1120
Circular thickness on standard pitch circle	.0785
Basic Rack Data:	
Flank angle	20
Tip to reference line	.0665
Tooth thickness at reference line	.0785
Tip radius	.0214

The test gears were assembled at a center distance of 2.0620 inches. This gave a nominal backlash of .0320 inches. This backlash permitted the test gears to reach relatively high torque levels without having the gear teeth roll back on each other, thereby making contact on the backside of the adjacent tooth. An illustration of the gear solid model assembly is shown in Figure 1.

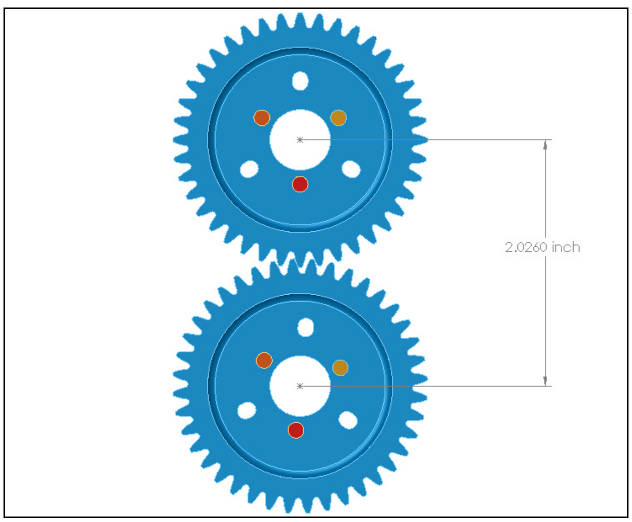

Figure 1

A parallel axis gear-testing machine developed by Ticona was used to load and record the load-displacement response of the gears. A schematic of the test machine is shown in Figure 2. The test gears were lubricated with oil prior to loading to eliminate any shearing forces acting on the tooth flanks that were in contact. Torque was measured on the stationary side and load was applied on the motor side. Two high precision encoders were used to measure the angular displacement of both gears. These encoders have a positional accuracy of 57600 counts per revolution. The rate of loading was set by the time for encoder position on the motor side. The stationary was not totally rigid. It required some angular displacement for the torquemeter to record data. To obtain the true angular displacement, the relative displacement between both gears was recorded. This gave a rate for the relative angular displacement between the motor gear and stationary gear to be about .002 radians per minute.

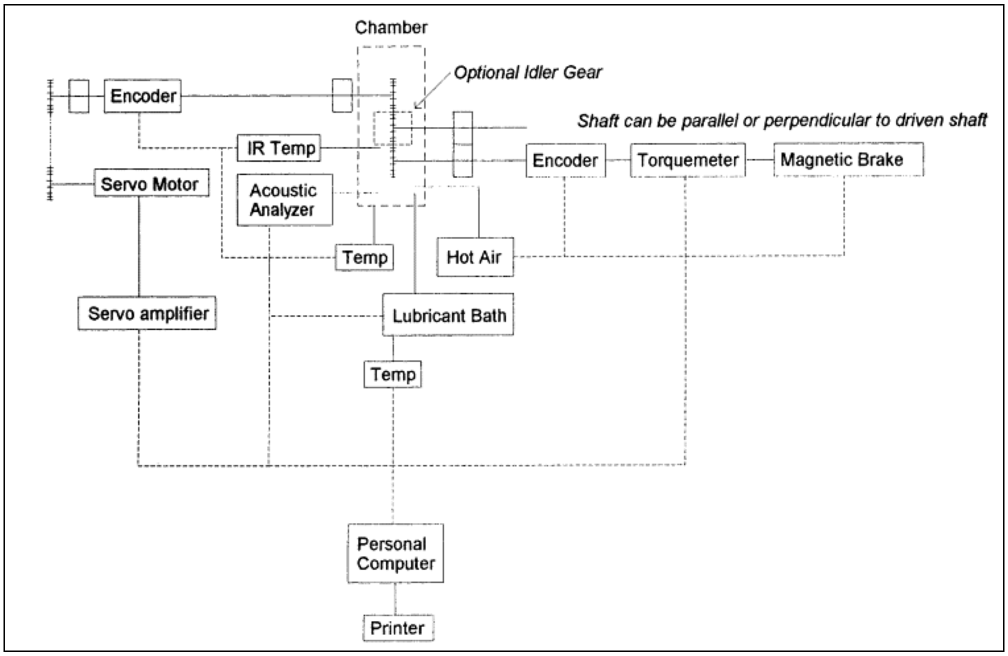

Figure 2

Five tests were made per gear set at ambient conditions. A plot of applied torques verses relative displacement was recorded. The results are shown in Figure 3. Test 2 and Test 4 did not reach tooth failure. This is due to that Test 4 was not taken up to the breaking torque and Test 2 reached the set limited encoder position before breaking.

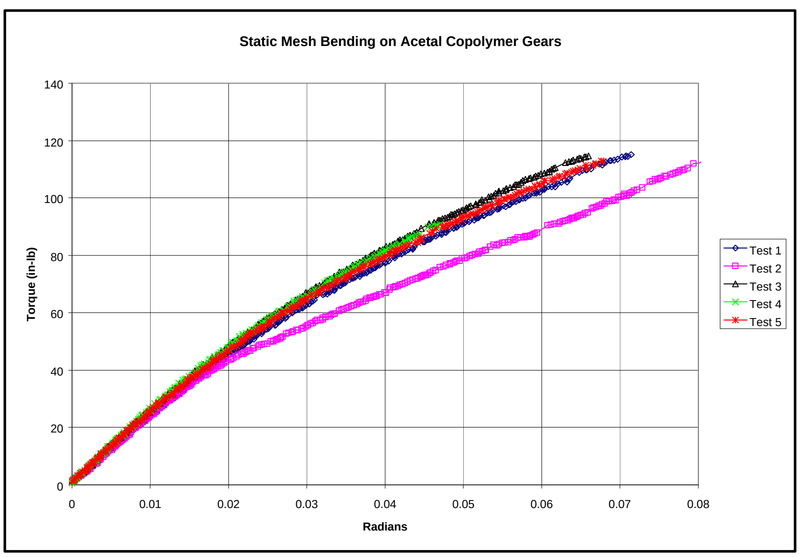

Figure 3

As can be seen in the Figure 3, with the exception of Test 2, the load verses displacement curve gave very repeatable results. Based on this, Test 1 was chosen as the representative experimental data to compare the elastic-plastic analysis with.

3. Analytical Approach

An elastic-plastic finite element analysis using the nonlinear package of MSC's MARC finite element code was used to model the deformation of the gear set under quasi-static conditions. The material response of the gears were based on the uniaxial stress-strain data obtain from physical testing for this material (see

Figure 4). Therefore, no viscous effects were considered in this analysis. That is, material response was considered to be time independent.

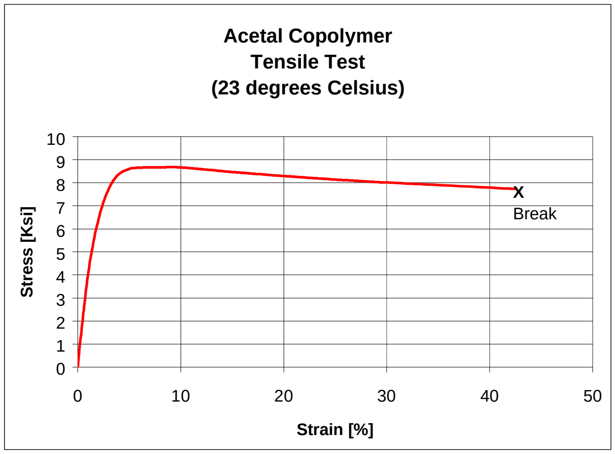

Figure 4

According to the International Standard for calculation of load capacity of spur and helical gears [1], the tooth root tensile stress has relevance to plane strain conditions. Therefore, the finite element analysis was performed assuming such a plane strain condition to exist on the gear set using MSC.Marc as shown in Figure 5.

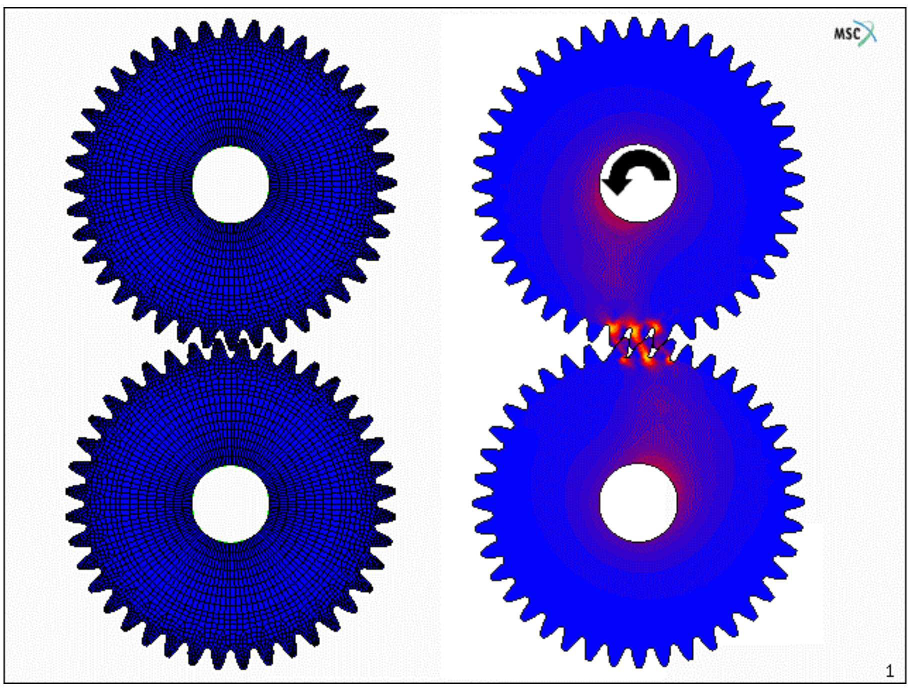

Figure 5

The full gear is modeled instead of a sector to capture the correct torsional stiffness of the gear as the top gear rotates into the stationary bottom gear. The original analysis simply increases the rotation of the top gear well into the plastic region of the material behavior. The torque rotation response of this model is shown in Figure 8, labeled “100% Strength Curve”. Because the simulation over shoots the maximum experimental torque, it was suspected that some material failure such as cracking was happening. To this end, two approaches were investigated for material failure.

The first approach to model material failure was base on deactivating the elements when the maximum tensile stress was reached. Elements that reached this value were removed during the simulation. An illustration of this can be seen in Figure 6. The green line shows the angular position that would of taken place for the gear teeth on the motor side (top gear) with respect to the gear teeth on the stationary (bottom gear) side without any deformation. Although this approach permitted loss of material to occur, it permitted tooth failure to follow it's own path by deactivating the elements that reached the maximum tensile stress. Here the failed teeth rolled back and made contact with the adjacent teeth.

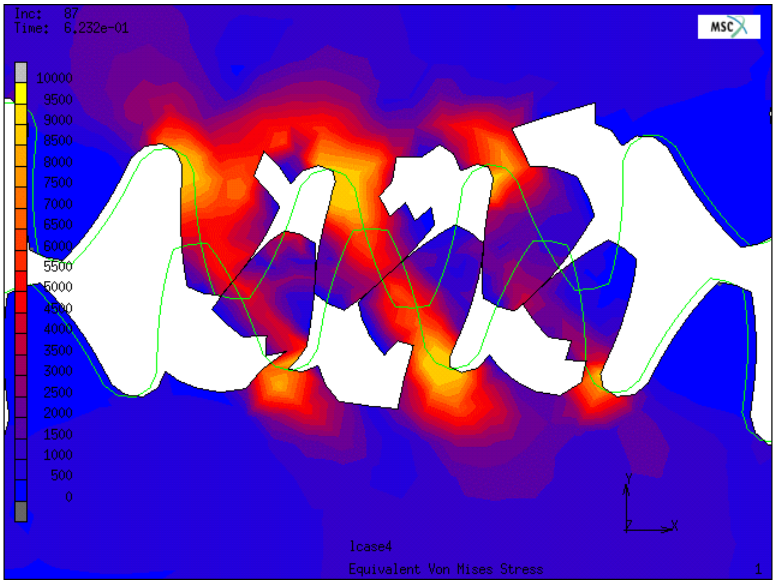

Figure 6

The second approach was based on permitting the elements to separate along a prescribed path when the maximum tensile stress was reach (see Figure 7). The advantage to this approach was that it did not permit material to be lost during tooth failure. However, unlike in the deactivated approach, the failure path had to be defined, as seen by the black lines underlining each tooth. Again, the magenta line shows the angular position that would of taken place for the gear teeth on the motor side (top gear) with respect to the gear teeth on the stationary (bottom gear) side without any deformation to have taken place.

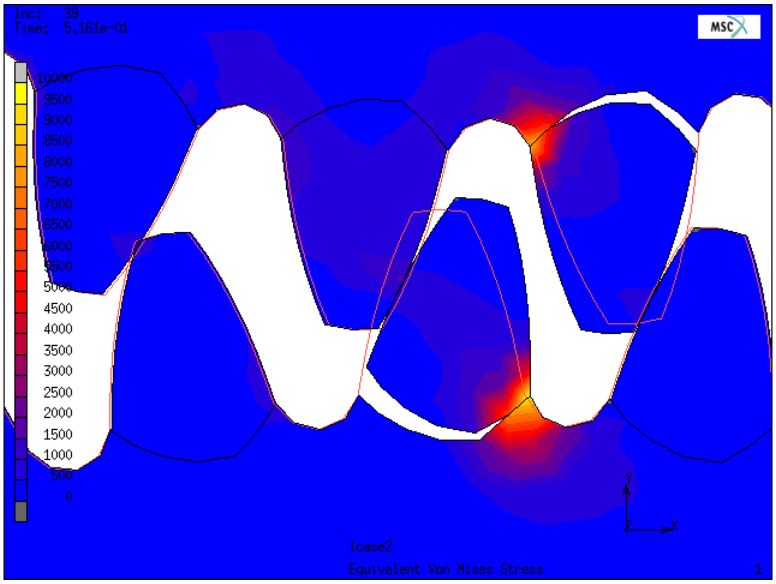

Figure 7

4. Comparison of Experimental and Analytical results

A plot of applied torque verses angular displacement was made using both the deactivating and separation of elements approach. As can be seen in Figure 8, at low angular displacements both deactivated and separation of elements give excellent represention of the experimental results. However, at the higher torque values, the deactivated approach gave more optimistic results, where as the separation approach was more conservative. In addition, it is seen that in both approaches, a loss of torque occurs when the tooth failure occurs. Because the deactivated approach requires a set of elements to be removed, it predicts a higher load capacity prior to loss of torque, where as in the separating approach, the elements are permitted to unzip once the maximum tensile stress is reached.

Although the separating of elements approach predicted the first tooth failure to occur at around 40 inch pounds of torque, and the second tooth failure at 60 inch pounds of torque, the slope of both those segments is very agreeable witht the experimental data. That is, if we were to remove the loss in torque section (i.e., neagative slope range) and only connect the remaining positive slope range, the plot woulbe be extremely accurate to the experimental data.

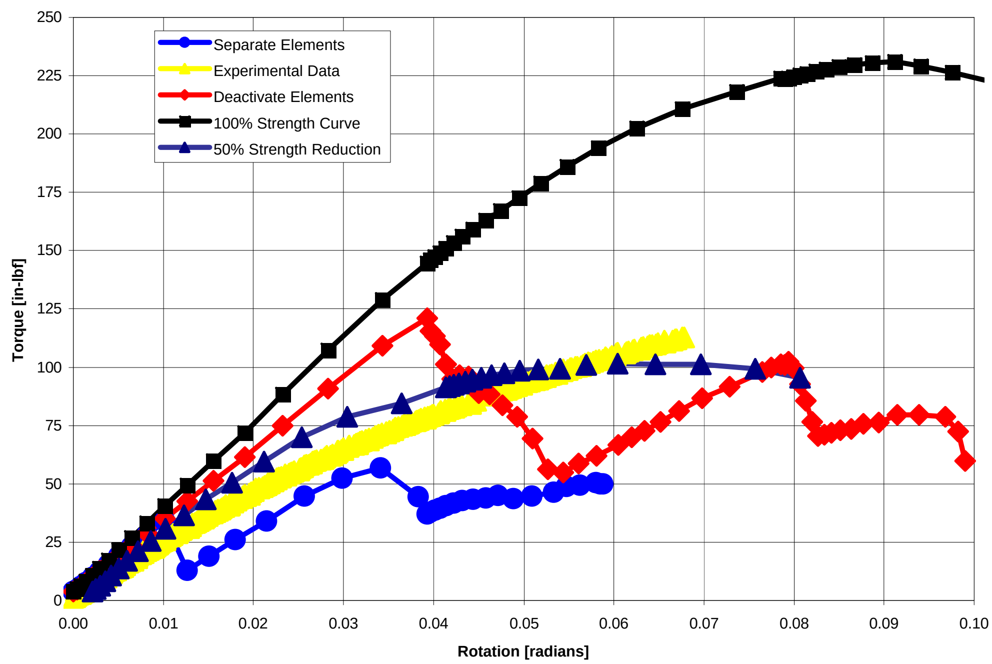

Figure 8

Based on those results, it was then attempted to conduct another analysis without permitting any material failure to occur. This was done by arbitraily using a 50% reduction of the stress data form Figure 4. As can be seen in Figure 8, the maximum torque and deformation coincide very well with the experimental results.

5. Discussion

Using the strength curve data, Figure 4, the finite element analysis over-estimates the maximum torque load as shown in Figure 8, Curve Title "100% Strength Curve". This is either due to fracture (material failure) or a non-typical strength curve for this material.

The material failure was then simulated by both separating and deactivating elements based upon a maximum tensile strength criterion. These simulations bound the experimental results nicely; the experimental results are bound above by deactivating elements and bound below (a conservative approach) by separating elements. With the prospect of material failure, the gear test were rerun with Test 2 and Test 4 of Figure 3 not run to failure. These tests indicated that material failure occurs just before failure (torque over 90 in-lbf), not before. This result prompted a revisit of the strength curve, suspecting that it may be too optimistic.

Assuming that the original strength curve was too optimistic, it was arbitraily reduced by 50% and the gear simulation was rerun with no material failure. The resulting torque-rotation response is very close to the experimental results. This has led to a re-examination of the tensile test used to determine the strength curve in Figure 4. Future tests are planned to re-examine the strength of this material. These tests would not only include uploading but also unloading data for specimens that are cut from the gears.

6. Conclusions

Based on the results of this analysis, the mechanical behavior and prediction of copolymer acetal gears is very complex. The results indicate that to optimize a gear set, a non-linear analysis is reqiured to be performed. Only under low loads and deformation can a linear elastic approach be suitable.

Clearly combining computer simulations with material and component testing has led to a far better understanding of copolymer acetal gear design; this understanding could not be achieved by either simulation or testing alone. It is envisioned that with a few more material tests, the torque-displacement response of the gear pair can be simulated with confidence thus advancing the technology of copollymer acetal gear applications.

7. References

1. Aida, Toshio and Terauchi Yoshio, “On the Bending Strength of a Spur Gear”, Bull of JSME, Vol. 5, No. 17, (1962), p.161-170

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3. Baronet, C.N. and Tordion G.V., “Exact Stress Distribution in Standard Teeth and Geometry Factors”, J. Engineering for Industry, Trans ASME, (Nov 1973), p. 1159-1163