Equibiaxial Stretching of Elastomeric Sheets, An Analytical Verification of Experimental Technique

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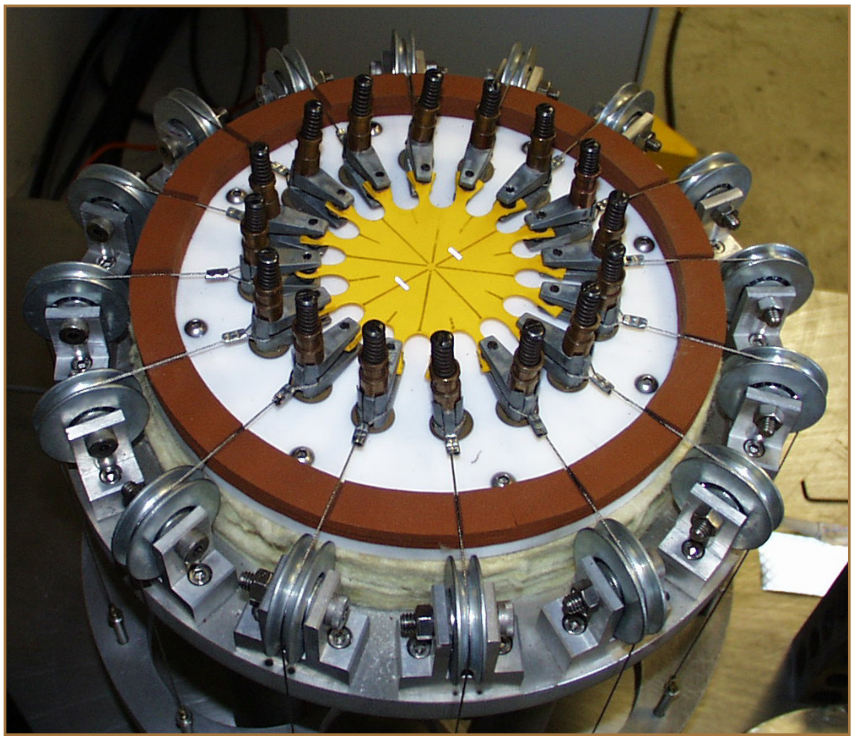

Figure 1, Biaxial stretching apparatus

Abstract
Constitutive models for hyperelastic materials may require multiple complimentary strain states to get an accurate representation of the material. One of these strain states is pure compression. Uniaxial compression testing in the laboratory is inaccurate because small amounts of friction between the specimen and the loading fixture cause a mixed state of compressive, shear and tensile strain. Since uniaxial compression can also be represented by equibiaxial tension, a test fixture was developed to obtain compressive strain by applying equibiaxial tensile loads to circular sheets while eliminating the errors due to friction. This paper outlines an equibiaxial experiment of elastomeric sheets while providing analytical verification of its accuracy.

Introduction

Constitutive models for hyperelastic materials are developed from strain energy functions and require nominal stress vs. nominal strain data to fit most models available. In general, it is desirable to represent the three major strain states which are; uniaxial tension, uniaxial compression, and pure shear.[1] If compressibility is a concern then bulk compressibility information is also recommended. The uniaxial tension strain state is easily obtained and the pure shear test can be performed using a planar tension test with excellent, repeatable accuracy. However, the uniaxial compression test is difficult to perform without introducing other strain states that will affect the accuracy. The main cause of the inaccuracy is the friction between the specimen and the loading platens. The friction can also vary as the compressive load (normal force) increases. To characterize the friction effect, an analysis of a standard ASTM D395, type 1 button under uniaxial compression loading was performed. A plot of compressive stress vs.

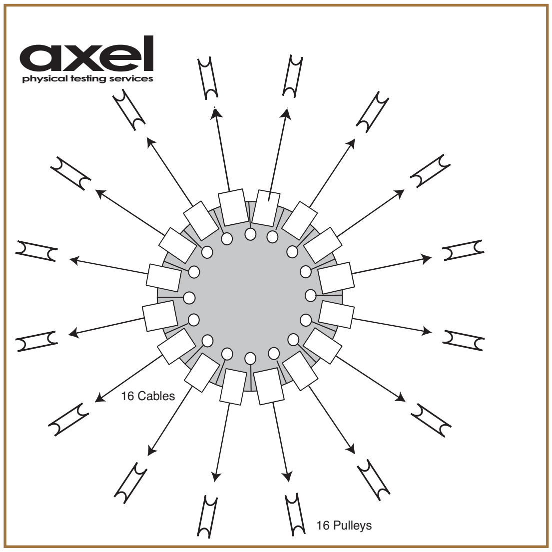

Figure 2, Biaxial Apparatus Schematic

compressive strain with varying coefficients of friction shows the variation caused by friction (see appendix A). The analysis of the standard button indicates that for small levels of friction the deviation from the pure uniaxial compressive strain state causes significant errors. An equibiaxial testing fixture is examined to determine if a pure compressive strain could be obtained accurately because an equibiaxial tension state of strain is equivalent to an uniaxial compressive strain.[12]

The equibiaxial straining apparatus described in this paper also has other advantages with respect to specimen availability and load control. These advantages include:

1. Achieving a strain condition equivalent to simple compression while avoiding the inherent experimental errors associated with compression.
2. Being able to perform strain and load control experiments as well as look at equilibrium behavior.
3. Testing on readily available test slabs.
4. Performing a test at the loading rates equivalent to tension and shear loading rates.
5. Performing tests at non-ambient temperatures.

Several other experimental approaches to the biaxial straining of elastomers have been developed[2-9]. In general, two approaches have been used. The first involves the expansion of a thin elastomer membrane using air pressure. Strain control is difficult to obtain with this procedure making it difficult to create conditions that compliment the other strains states required to get a full set of data for fitting hyperelastic constitutive equations. The other problem is that the thickness of the sheets needs to be much thinner than the typical sheet thickness that is created. The second approach involves the gripping of a rectangular specimen around the perimeter and stretching the specimen with multiple arms or cable bearing systems. This approach has been used with great success by several investigators. Difficulties arise with the measurement of strain and the calculation of stress. The advantage of this approach is that while somewhat complex, it allows the investigator to examine elastomer deformation in unequal biaxial deformation states.

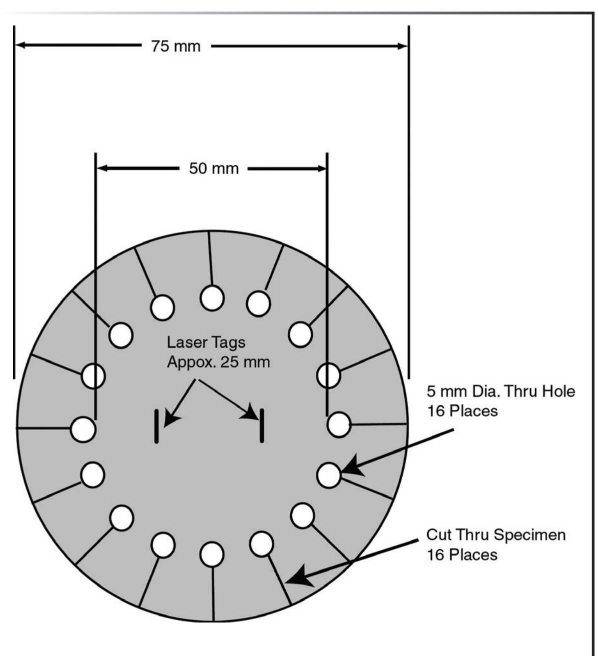

Figure 3, Biaxial Test Specimen Outline

Since the objectives herein do not involve the need for unequal biaxial straining, the mechanical aspects of the experimental approach can be greatly simplified and the relations between forces and stresses in the specimen can be ascertained with greater certainty by restricting the apparatus to equal biaxial straining.

Overall Approach

The overall approach is to strain a circular specimen radially. Constant stress and strain around the periphery of the disk will create an equibiaxial state of stress and strain in the disk independent of thickness or radial position[11].

The Experimental Apparatus

Applying Radial Forces

In the apparatus, 16 small grips mechanically attach to the perimeter of an elastomer disk using spring force attachment. The grips are moved radially outward by pulling with thin flexible cables which are redirected around pulleys to a common loading plate (Figure 1). When the loading plate is moved all of the attachment points move equally in a radial direction and a state of equal biaxial strain is developed in the center of the disk shaped specimen (Figure 2).

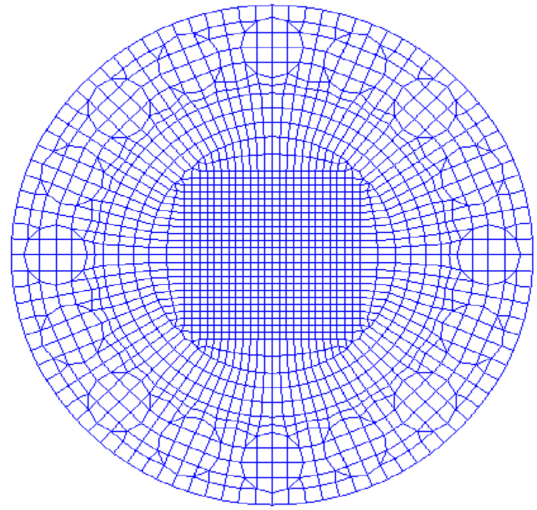

Figure 4, FEA model of uncut specimen with radial loads applied at every perimeter node.

The Specimen

The actual shape of the specimen is not a simple disk as shown in Figure 3. There are radial cuts introduced into the disc specimen so that there are no tangential forces between the grips. This is necessary because the grips are not attached to the outer edge of the specimen. They are attached to the top and bottom surfaces of the specimen which does not allow material to flow within the grip. Small holes are introduced at the ends of the radial cuts so that the specimen is less likely to tear.

Strain Measurement

The relationship between grip travel and actual straining in the center area of the specimen is not known with certainty because of the unknown strain field around the grips and the compliance that may exist in the loading cables and the material flowing from the grips. To determine the strain, a laser non-contacting extensometer is used to measure the strain on the surface of the specimen away from the grips.

Force Measurement

The total force transmitted by the 16 grips to the common loading plate is measured using a strain gage load cell.

Relating Force Measured to Stress

The nominal equibiaxial stress contained inside the specimen inner diameter (D_i) is calculated as follows:

$$\sigma = F/(\pi * D_i * t)$$

where: D_i = Diameter as measured between punched holes

F = Sum of radial forces

t = Original thickness

σ = Engineering stress

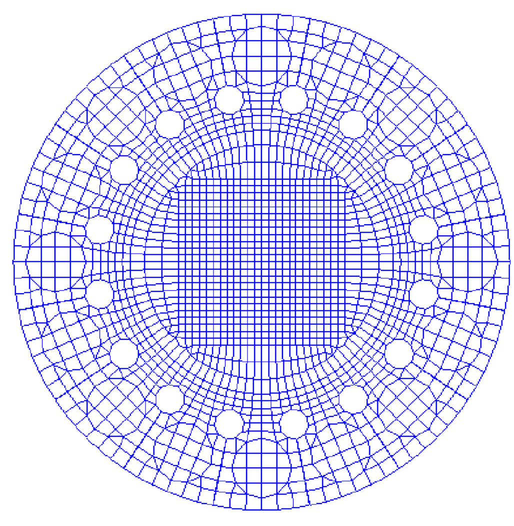

Figure 5, FEA model of specimen with slits and punched holes, radial loads applied at 16 grip locations.

Analytical Verification

Once the closed form solution has shown that a circular disk pulled with a uniform circumferential load produces a biaxial stress and strain field we then need to verify that pulling the disk from 16 discrete grip locations is an acceptable approximation.

The following analytical procedure will examine the effects of the boundary conditions imposed by the experimental approach on the ideal closed form solution. The experimental aspects of concern are:

A. The specimen is not gripped continually around the circumference.

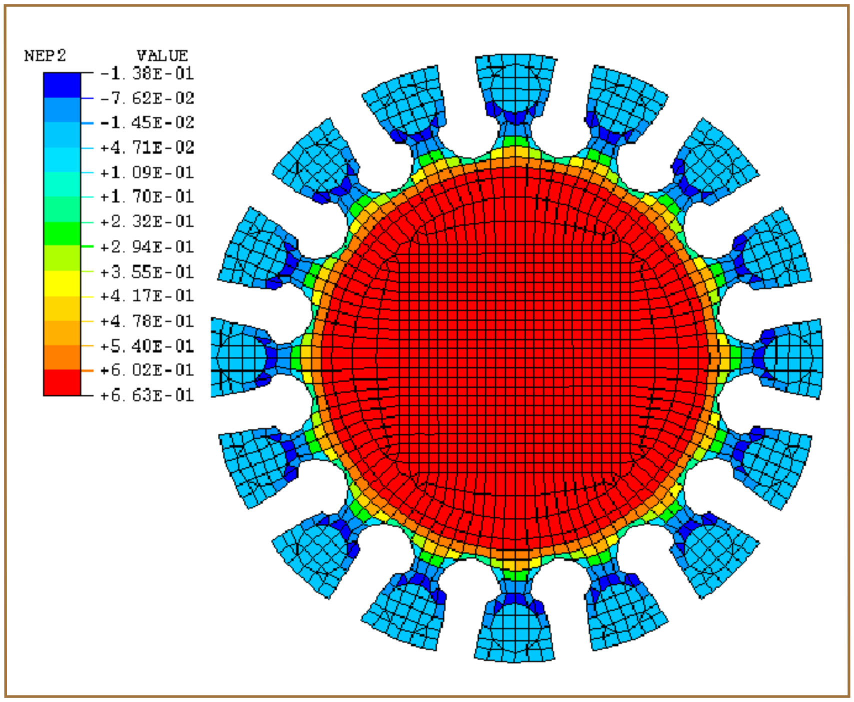

Figure 6, Specimen Deformed Shape

B. Cuts are introduced between the grips that alter the strain field.

C. The relationship between force and stress is based on the "inside" diameter indicated in Figure 3.

First finite element analysis is used to verify the closed form solution on a representative specimen model. The following steps will show how the proposed specimen will be compared to the closed form solution.

Closed Form Solution Comparison

The disk specimen finite element model used to verify the closed form solution is shown in Figure 4. Radial loads are applied at every node around the perimeter. The nominal finite element stress calculated within each element was compared to the stress calculated with the formula below and found to be equivalent.

$$\sigma = F/(\pi * D * t)$$

where: F = force (sum of radial forces)

D = original outside diameter

t = original thickness

σ = engineering stress

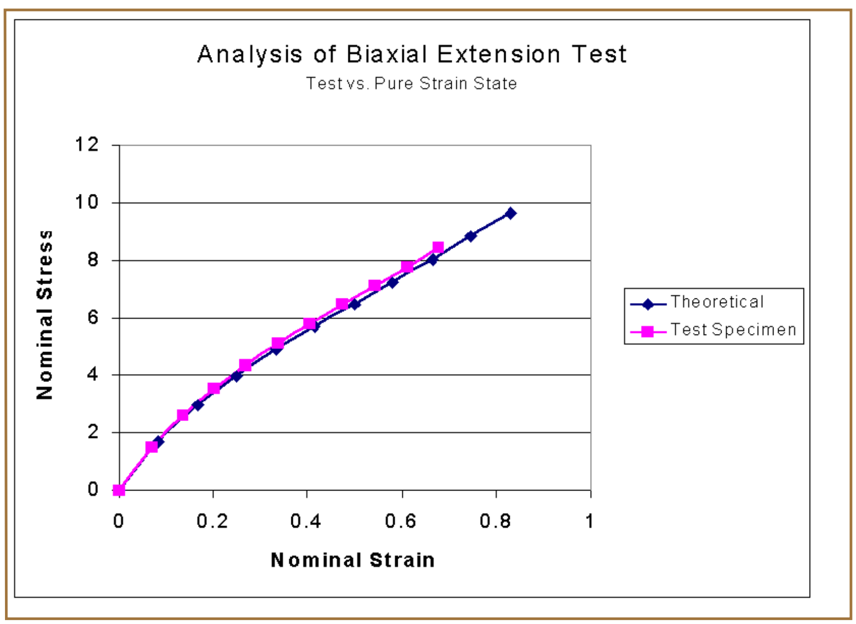

Figure 7, 2nd Order Polynomial Fit

This formula can now be used in a testing environment since all the parameters are known.Analysis of the Experimental Condition

The next step needs to show that using a cut specimen with 16 grips (FEA model shown in Figure 5) will accurately represent the "ideal" loading condition of the previous finite element analysis. The original outside diameter used in the above stress formula will be equal to the diameter measured at the inside edges of the punched holes at the ends of the radial slits between the grips. For the proposed configuration this dimension is 50 mm. A deformed shape sequence of this configuration under loads is shown in Figure 6. A nominal stress vs. nominal strain comparison of this configuration vs. FEA "closed form" results is shown for two

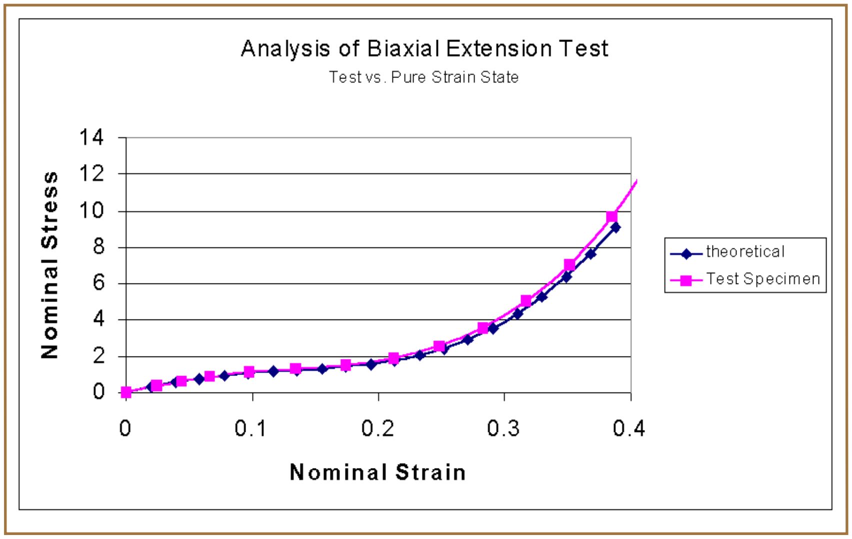

Figure 8, 5-term Ogden Fit

hyperelastic material representations. The first (Figure 7) represents a simple 2nd order polynomial approximation and the second (Figure 8) represents an Ogden 5-term approximation. Both show excellent correlation between the proposed test configuration and the theoretical results.

Summary

The equibiaxial experiment as proposed in this paper does an excellent job of obtaining the pure strain state required for hyperelastic constitutive models. The error due to the boundary condition approximations are small but consistent as opposed to the uniaxial compression test where the experimental error depends on friction which is unknown and varies as a function of the test material and the normal force. The testing done in this manner can provide excellent consistant and accurate compression strain states while using standard ASTM slabs and a minor amount of specimen preparation to perform.

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